**Mathematical modelling**

**Program**

**1. Introduction. Mathematical models of flight dynamics**

General principles of mathematical modelling were discussed. The structure of mathematical models was described. A classification of mathematical models was given. As an example, we considered the problem of a body falling under its own weight.

The subject of Task 1 is mathematical models of flight dynamics, which are generalizations of the mathematical model of the body falling process. In the following tasks, it is required to indicate the marked characteristics of the corresponding mathematical model, just as it was done in lecture for the model of the body falling.

**2. Mechanical oscillations. Oscillations of pendulum and spring**

Mathematical model of the process of pendulum oscillations was considered. In particular, there is given the derivation of the equation for the oscillation of the pendulum, its solution is given, the law of conservation of the oscillation energy is derived, the equilibrium position of the system is established, mathematical models of the oscillation of the pendulum in the presence of friction, as well as under the action of an external force, are considered.

The first six variants of the task relate to the energy characteristics and the equilibrium position of the pendulum in the presence of friction. When performing these tasks, one should be based on the formulas given in the lecture for calculating the kinetic and potential energy and the concept of the equilibrium position of a pendulum in the absence of friction considered in the lecture.

The rest of the tasks are associated with a mathematical model of the spring oscillation process. The derivation of this mathematical model is given in the Appendix to Lecture 2. When completing the task, one should start from the analogy between the model of pendulum oscillation described in detail in the lecture and the model of spring oscillation considered in the task.

In all tasks, it is required not only to give the corresponding results, but also indicate their physical interpretation.

**3. Electrical oscillations. Oscillations of electrical circuit**

Mathematical models of processes associated with an electrical circuit were considered. The main result here is the presence of a deep analogy between the mechanical processes associated with the movement of a pendulum and a spring, and the behavior of an electrical circuit. Mathematically, these processes are identical, being described by the same equations. When performing the next task below, one should be based on this analogy.

In fact, any task can be completed in three stages:

1. Translation of the delivered task from the electrical language to the mechanical one.

2. Using the already known corresponding result from the previous lecture.

3. Reverse translation of the result obtained from mechanical language into electrical one.

**4. Mathematical model of chemistry. Chemical kinetic problems**

Mathematical models of chemical processes were considered, which characterize the change over time of the initial substances and products of chemical reactions. In the following tasks, it is required to write down a mathematical model for a specified system of chemical reactions, which is a system of differential equations for all substances participating in the reactions, with the corresponding initial conditions. As a sample, here is a mathematical model for the Lotka reaction system. Indicate the order of each of the considered reactions.

**5. Mathematical model of chemistry. Symbiosis model**

Mathematical models of biological processes that characterize the change over time in the number of biological species under different conditions of their existence were considered. In the following tasks, it is required for the symbiosis model to select specific numerical values of all parameters of the system at which the specified effect is realized and to explain the results obtained from the point of view of biology. In a number of variants, the described situation is impossible. In this case, the reason for the impossibility of the situation should be explained.

**6. Mathematical model of economics. Competition model**

Mathematical models of economic processes were considered. The purpose of this assignment is to study a **competition model** that also has biological meaning. It is required to explain the meaning of this combination of parameters and to describe the development of events from the initial state of the system to the end of the process. All explanations should be carried out in both **economic** and **biological** interpretation.

**7. Mathematical model of social sciences. Niche model**

Mathematical models in the social sciences. The purpose of this assignment is to study a niche model that also has biological and economic meaning. In the task, it is required to select specific parameters of the system (equation coefficients and initial states) in the variables *u*, *v* so that the effect described in the task is observed. Describe the corresponding evolution of the system in the interpretation specified in the assignment.

**8. Heat equation**

Mathematical models of transfer processes were considered, which are the first and second boundary value problems for a homogeneous and inhomogeneous heat equation. In the following tasks, specific conditions for the flow of the heat process in a thin homogeneous body of a given length are described

The following steps are required:

1. Write a mathematical model of the process.

2. Using ready-made formulas for solutions from the text of a lecture or presentation, give a solution to the set boundary value problem.

3. Make sure that this is indeed a solution to the problem by substituting it into the equation and boundary conditions.

4. Indicate how and why there is a change in body temperature with time at its various points, taking as a sample the corresponding descriptions from the lecture.

**9. Transfer processes**

Mathematical models of various transfer processes were discussed. The following tasks describe the specific conditions for a complex transfer process, when, on the one hand, there is a certain chemical, biological or economic process considered before in the first part of the course, and, on the other hand, events occur in a certain one-dimensional area, as a result of which the corresponding process is implemented transfer. It is required to give a complete mathematical model of the process, including a system of state equations with appropriate initial and boundary conditions.

**10. Wave processes**

Mathematical models of string vibrations were considered, which are the first (fixing the ends of the string) and the second (free ends of the string) boundary value problems for the corresponding homogeneous equation.

The following steps are required:

1. Write down a mathematical model of the process.

2. Using ready-made formulas for solutions from the text of a lecture or presentation, give a solution to the boundary value problem.

3. Make sure that this is indeed a solution to the problem by substituting it into the equation and boundary conditions.

4. Explain how and why vibrations of a string occur with time at its various points, taking as a model the corresponding descriptions from the lecture. Consider the position an velocity of different points of string.

**11. Field theory**

Mathematical models of electrostatic and gravitational fields were considered. In this case, the potentials of these fields are described by the Poisson equation. It is known that the electrostatic field in the absence of charges and the gravitational field in vacuum are described by the Laplace equation.

Tlectrostatic or gravitational field in three-dimensional space is considered in the case of spherical or cylindrical symmetry. The point where the point source is located, or the straight line corresponding to the direction of the wire, is indicated. The charge (for the wire - the charge density) of the source of the gravitational field or the mass of the source of the gravitational field is known.

The following steps are required:

1. Write down the equation for the field potential with the indicated type of symmetry.

2. Using ready-made formulas for solutions from the text of a lecture or presentation, give a solution to the set boundary value problem.

3. Make the change of variables by placing a point source at the origin or by pointing the wire along the *z* axis.

4. Find the value of the potential of the corresponding field at the point specified in the task.

5. Comment on the results.

**12. Variational principles**

Various issues related to the application of variational principles. It is necessary to determine movement equations using variational principles.

**13. Discrete models**

Some problems of game theory are considered. It is necessary to determine Nash equilibrium and Pareto optimality.